

# Quark Confinement Physics in Quantum Chromodynamics

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We study abelian dominance and monopole condensation for the quark confinement physics using the lattice QCD simulations in the MA gauge. These phenomena are closely related to the dual superconductor picture of the QCD vacuum, and enable us to construct the dual Ginzburg-Landau (DGL) theory as an useful effective theory of nonperturbative QCD. We then apply the DGL theory to the studies of the low-lying hadron structure and the scalar glueball properties.

## 1. Introduction

Recent studies of the lattice QCD in the maximally abelian (MA) gauge suggest the remarkable properties of the QCD vacuum, such as abelian dominance[1] and monopole condensation[2], which provide the dual superconductor picture of the QCD vacuum as is described by the dual Ginzburg-Landau (DGL) theory[3]. In the MA gauge, QCD is reduced into an abelian gauge theory including color-magnetic monopoles. According to the lattice QCD results, the nonperturbative quantities as the string tension and the chiral condensate are almost reproduced only by the diagonal gluon part, while the off-diagonal gluon does not contribute to such the long-range physics, namely, abelian dominance. Furthermore, the world-line of the color-magnetic monopole in the confinement phase appears as the global network, which indicates monopole condensation. Then, the DGL theory can be constructed by extracting the diagonal gluon as the relevant degrees of freedom and taking into account monopole condensation. Based on the DGL theory, the quark confinement is explained by the flux-tube formation through the dual Meissner effect, and chiral symmetry breaking is described as the function of monopole condensate[3].

In this paper, we focus such the dual superconductor picture of the QCD vacuum in the MA gauge, and confirm the connection between nonperturbative QCD and the DGL theory. Then, we would like to apply the DGL theory to hadron physics, especially, to the analysis of the scalar glueball properties.

## 2. Abelian dominance and monopole condensation in the MA gauge

Abelian dominance and monopole condensation in the MA gauge are the keywords to connect the QCD with the DGL theory, and the recent lattice QCD simulations show the former on the string tension and the chiral condensate, and the latter as the large clustering of the monopole world-line. In such situation, we still have interest in this subject, since the physical essence of abelian dominance is not understood yet. Furthermore, we

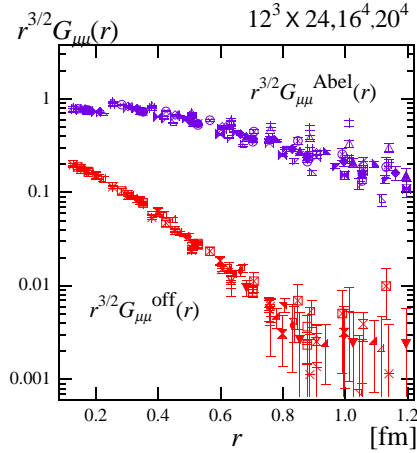


Fig. 1. The gluon propagator as a function of 4-dimensional distance  $r$  in the MA gauge.

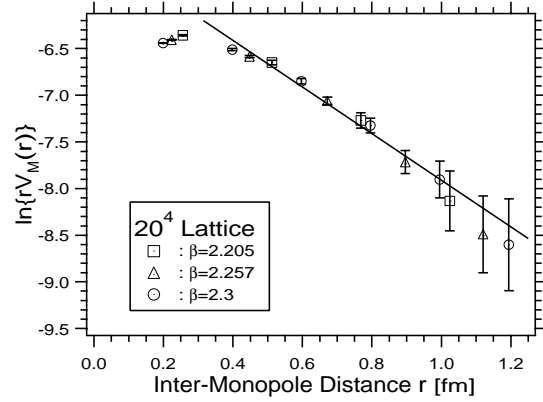


Fig. 2. The inter-monopole potential as a function of 4-dimensional distance  $r$  in the MA gauge.

must not jump to a conclusion that the global network of the monopole world-line is really the evidence of monopole condensation, which also should be evaluated quantitatively.

To answer these questions, we first study the gluon propagator in the MA gauge and evaluate the mass of the off-diagonal gluon field using the SU(2) lattice QCD simulation[4]. This study is based on the following idea. If the off-diagonal gluon has a mass such as the massive vector boson, its propagator  $G_{\mu\mu}^{\text{off}}(r)$  would be described by the Yukawa-type function  $\sim \exp(-M_{\text{off}}r)/r^{3/2}$ , and if we find the linear behavior for  $\ln(r^{3/2}G_{\mu\mu}^{\text{off}}(r))$ , the mass  $M_{\text{off}}$  can be extracted from its slope. As a result, we find that the off-diagonal gluon has the large mass  $M_{\text{off}} \simeq 1$  GeV as shown in Fig. 1. That is to say, the interaction range of the off-diagonal gluon is limited within the short distance corresponding to its inverse mass  $M_{\text{off}}^{-1} \simeq 0.2$  fm. Thus, the off-diagonal gluon does not contribute to the long-range physics, which predicts general infrared abelian dominance in the MA gauge.

As for monopole condensation, we study the inter-monopole potential and evaluate the dual gluon mass using the SU(2) lattice QCD simulation[5]. The dual gluon field  $B_\mu$  is introduced to satisfy  $\partial_\mu B_\nu - \partial_\nu B_\mu = *F_{\mu\nu}$  and  $\partial_\mu *F_{\mu\nu} = k_\nu$ . Here,  $k_\nu$  is the color-magnetic monopole current. The idea used here is quite similar to the evaluation of the off-diagonal gluon mass. If monopole condensation is occurred, the dual gluon becomes massive due to the dual Higgs mechanism. Then, its mass  $m_B$  can be extracted by fitting the Yukawa potential  $V_M(r) \sim -\exp(-m_B r)/r$ , since the dual gluon behaves as the massive vector boson. From this analysis, we find that the dual gluon acquires the mass  $m_B \simeq 0.5$  GeV as shown in Fig. 2, which is just the quantitative evidence of monopole condensation.

As an interesting application of abelian dominance for the inter-quark potential, we can calculate the quark single-particle potential  $U(x)$  for the low-lying hadron ( $m_q=300$  MeV). Here,  $U(x)$  is defined by the superposition of the inter-quark potential  $V(r) = -c/r + \sigma r$  ( $\sigma \simeq 1$  GeV/fm,  $c \simeq 0.4$ ) with the weight of the color charge distribution  $\rho(\mathbf{x}) = \bar{\psi}_q \gamma_0 \vec{H} \psi_q \cdot \vec{Q}$  as  $\vec{Q}^2 U(x) = \int d^3x' \rho(\mathbf{x}') V(|\mathbf{x} - \mathbf{x}'|)$ . Solving the self-consistent equations between the quark wave function and the quark potential, we obtain the color charge distribution and the quark single-particle potential as shown in Figs. 3 and 4. The color charge distribution is spread over a intermediate region  $r \sim 0.5$  fm. The quark single-particle potential is found to be flat at the short distance, which can be connected with the bag model.

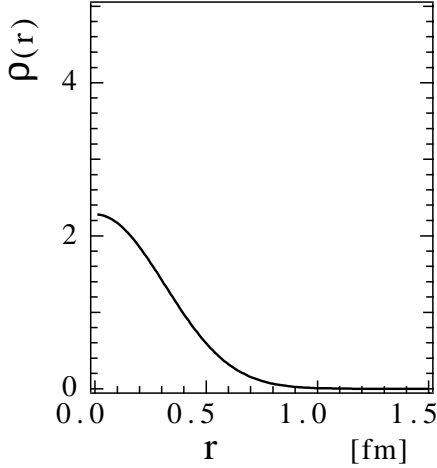


Fig. 3. The color charge distribution for the low-lying hadron.

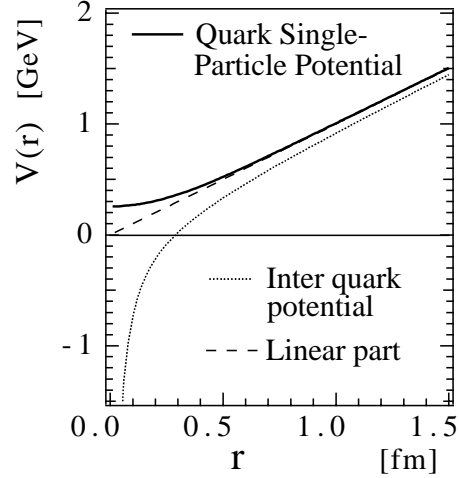


Fig. 4. The quark single particle potential in the low-lying hadrons.

### 3. The DGL theory and application to the scalar glueball analysis

The DGL theory can be constructed by taking into account abelian dominance and monopole condensation in the MA gauge in QCD. The DGL lagrangian is given as

$$\mathcal{L}_{\text{DGL}} = -\frac{1}{4} \left( \partial_\mu \vec{B}_\nu - \partial_\nu \vec{B}_\mu - \frac{1}{n \cdot \partial} \varepsilon_{\mu\nu\alpha\beta} n^\alpha \vec{j}^\beta \right)^2 + \sum_{\alpha=1}^3 \left[ \left| (\partial_\mu + ig \vec{\epsilon}_\alpha \cdot \vec{B}_\mu) \chi_\alpha \right|^2 - \lambda (|\chi_\alpha|^2 - v^2)^2 \right], \quad (1)$$

where  $\vec{B}_\mu$  and  $\chi_\alpha$  denote the dual gluon field with two components ( $B_\mu^3, B_\mu^8$ ) and the complex scalar monopole field, respectively. The quark field is included in the current  $\vec{j}_\mu = e \bar{q} \gamma_\mu \vec{H} q$ . Here,  $\vec{\epsilon}_a$  is the root vector of SU(3) algebra, and  $n^\mu$  denotes an arbitrary constant 4-vector, which corresponds to the direction of the Dirac string. The gauge coupling  $e$  and the dual gauge coupling  $g$  hold the relation  $eg = 4\pi$ .

Monopole condensation is characterized by  $\langle 0 | \chi_\alpha | 0 \rangle = v$ , and the dual gluon field acquires the mass  $m_B = \sqrt{3}gv \simeq 0.5$  GeV through the dual Higgs mechanism. Then, the DGL theory describes the QCD vacuum as the dual superconductor. The quark confinement is explained by the dual Meissner effect, which forces the color-electric field between the quarks to form the flux-tube configuration, and leads the linear inter-quark potential. This flux-tube also provides intuitive picture of hadrons. If we apply this flux-tube picture to the glueball, it would be identified with the flux-tube ring, since the glueball is considered to have no valence quarks, and the lowest state is the scalar glueball. From the flux-tube ring solution in the DGL theory, we find the mass and the size of the scalar glueball as 1.6 GeV and 0.5 fm, respectively[6]. It is interesting to note that this mass spectrum is consistent with the recent lattice QCD results  $M(0^{++}) = 1.50 - 1.75$  GeV[7].

Here, we find another aspect of the scalar glueball in the DGL theory, which is closely related to the dual Higgs mechanism. Taking monopole condensation into account, the monopole field can be defined as  $\chi_\alpha \equiv \left( v + \tilde{\chi}_\alpha / \sqrt{2} \right) e^{i\eta_\alpha/v}$ , where  $\tilde{\chi}_\alpha$  and  $\eta_\alpha$  are real variables denoting the magnitude of the vacuum fluctuation and the phase, respectively. Here,  $\alpha=1, 2, 3$  labels the color-magnetic charge of the monopole field, dual-red, dual-blue and dual-green. Since the origin of the monopole field is the off-diagonal gluon field in the MA gauge in QCD, this field  $\tilde{\chi}_\alpha$  would present the scalar gluonic excitation corresponding

to the dual Higgs particle. In particular, the Weyl symmetric monopole field defined by  $\tilde{\chi}^{(0)} \equiv (\tilde{\chi}_1 + \tilde{\chi}_2 + \tilde{\chi}_3)/\sqrt{3}$  is the color-singlet field[8] so that it can be regarded as the scalar glueball with the mass  $m_\chi = 2\sqrt{\lambda}v \simeq 1.6$  GeV. Although the relation between the flux-tube ring is not clear, it can be considered as another feature of the scalar glueball.

Here, we concentrate on the calculation of the  $\tilde{\chi}^{(0)}q\bar{q}$  vertex function, which plays an important role to understand how the scalar glueball interacts with the quarks. The lowest diagram is shown in Fig. 5. The scalar glueball interacts with the dual gluon at first, and then, the dual gluon interacts with the quarks. We show the typical behavior of the vertex function in the scalar channel, as a function of the coupled quark momentum in Fig. 6. Here, we have set  $p \cdot q = 0$  for simplicity. We find that the heavy quark ( $m_c \simeq 1.6$  GeV) interacts with the scalar glueball about four times stronger than the light quarks ( $m_{u,d,s} \simeq 0.3 - 0.5$  GeV). It seems to indicate the flavor dependence of the interaction of the scalar glueball. It is interesting to study how this interaction property reflects on the scalar glueball decay into the two pseudo-scalar mesons and the glueball-quarkonium mixing states, which are now investigating.

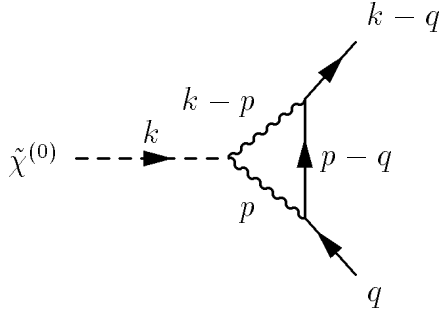
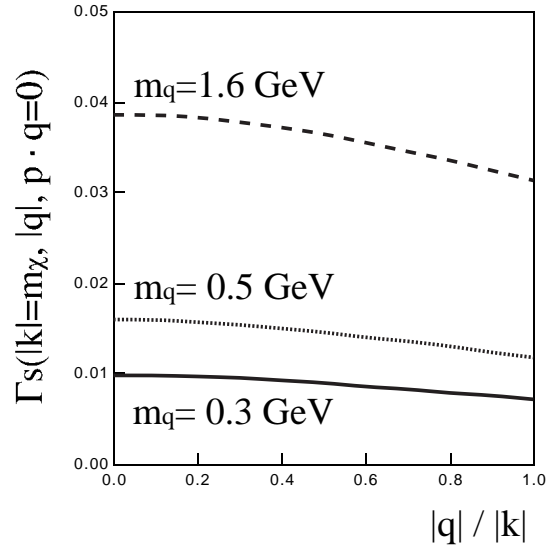


Fig. 5(upper).  $\tilde{\chi}^{(0)}q\bar{q}$  vertex.

Fig. 6(right). The vertex function of  $\tilde{\chi}^{(0)}q\bar{q}$  in the scalar channel vs. coupled quark momentum.



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